$$
\begin{array}{rcl}
A_0 & = & \{\varnothing\} \\
A_{n+1} & = & \{\{(\alpha_i,\beta_i)\}_{i\in k} \mid k\in\omega\setminus 1 \wedge \alpha_i \in \mathrm{Ord}\setminus 1 \wedge \beta_i \in A_n \wedge |\{\beta_i\}_{i\in k}| = k\} \\
A & = & \bigcup\limits_{n\in\omega}(A_n) \\
A' & = & A\setminus\{\varnothing\}
\end{array}
$$

$$
\begin{array}{rcl}
B_0 & = & \{\varnothing\} \\
B_{n+1} & = & \{\{(\alpha_i,\beta_i)\}_{i\in k} \mid k\in\omega\setminus 1 \wedge \alpha_i \in \mathrm{Ord} \wedge \beta_i \in B_n \wedge |\{\beta_i\}_{i\in k}| = k\} \\
B & = & \bigcup\limits_{n\in\omega}(B_n)
\end{array}
$$

$$
\begin{array}{rcccl}
a & : & A & \to & \bigcup\limits_{n\in\omega}(\{\{\alpha_i\}_{i\in n,\alpha_i\in\mathrm{Ord}}\}) \\
a(X) & : & X & \mapsto & \{X' \mid \exists X'' \in (\mathrm{Ord}\setminus 1)[(X'',X')\in X]\} \\
b & : & A' & \to & A \\
b(X) & : & X & \mapsto & \hat{X}'(\nexists X'' \in a(X)[X' \prec X'']) \\
\overline{b} & : & A' & \to & A \\
\overline{b}(X) & : & X & \mapsto & \hat{X}'(\nexists X'' \in a(X)[X'' \prec X'])
\end{array}
$$

$$
\begin{array}{rclcll}
c_X(Y) & : & A^2 & \to & \mathrm{Ord} & \\
c_X(Y) & : & (X,Y) & \mapsto & \hat{X}'((X',Y)\in X) & \iff Y\in a(X) \\
 & & (X,Y) & \mapsto & 0 & \iff Y\notin a(X)
\end{array}
$$

$$
X \prec Y \iff \begin{cases}
\bot & [Y=\varnothing] \\
\top & [X=\varnothing \wedge Y\neq\varnothing] \\
b(X)\prec b(Y) & [X\neq\varnothing \wedge Y\neq\varnothing \wedge b(X)\neq b(Y)] \\
c_X(b(X)) \prec c_Y(b(Y)) & [X\neq\varnothing \wedge Y\neq\varnothing \wedge b(X)=b(Y) \wedge c_X(b(X))\neq c_Y(b(Y))] \\
X\setminus\{(c_X(b(X)),b(X))\} \prec Y\setminus\{(c_Y(b(Y)),b(Y))\} & [X\neq\varnothing \wedge Y\neq\varnothing \wedge b(X)=b(Y) \wedge c_X(b(X)) = c_Y(b(Y))]
\end{cases}
$$

$$
\begin{array}{rcccl}
d & : & B & \to & A \\
d(X) & : & X & \mapsto & \{(X',X'') \mid (X',X'')\in X \wedge X' > 0\} \\
e & : & A & \to & A
\end{array}
$$

$$
e(X) : X \mapsto \begin{cases}
X & [c_X(\varnothing)\in \mathrm{Lim}\cup 1] \\
d(X\setminus\{(c_X(\varnothing),\varnothing)\}\cup\{(\hat{\alpha}(\alpha+1=c_X(\varnothing)),\varnothing)\}) & [c_X(\varnothing)\in\mathrm{Succ}]
\end{cases}
$$

$$
f : A \to \mathrm{Ord}
$$

$$
f(X) : X \mapsto \begin{cases}
0 & [X=\varnothing] \\
1 & [e(X)\neq X] \\
2 & [e(\overline{b}(X))\neq \overline{b}(X) \wedge c_X(\overline{b}(X))\in\mathrm{Succ}] \\
f(\overline{b}(X)) & [e(\overline{b}(X)) = \overline{b}(X) \wedge c_X(\overline{b}(X))\in\mathrm{Succ}] \\
c_X(\overline{b}(X)) & [c_X(\overline{b}(X))\in\mathrm{Lim}]
\end{cases}
$$

$$
g : A\times\mathrm{Ord} \to A
$$

$$
g(X,\alpha) : (X,\alpha) \mapsto \begin{cases}
\varnothing & [X=\varnothing] \\
e(X) & [e(X)\neq X] \\
d(X\setminus\{(c_X(\overline{b}(X)),\overline{b}(X))\}\cup\{(\hat{\beta}(\beta+1=c_X(\overline{b}(X))),\overline{b}(X)),(\alpha,e(\overline{b}(X)))\}) & [e(\overline{b}(X))\neq\overline{b}(X) \wedge c_X(\overline{b}(X))\in\mathrm{Succ}] \\
d(X\setminus\{(c_X(\overline{b}(X)),\overline{b}(X))\}\cup\{(\hat{\beta}(\beta+1=c_X(\overline{b}(X))),\overline{b}(X)),(1,g(\overline{b}(X),\alpha))\}) & [e(\overline{b}(X)) = \overline{b}(X) \wedge c_X(\overline{b}(X))\in\mathrm{Succ}] \\
d(X\setminus\{(c_X(\overline{b}(X)),\overline{b}(X))\}\cup\{(\alpha,\overline{b}(X))\}) & [c_X(\overline{b}(X))\in\mathrm{Lim}]
\end{cases}
$$

$$
h : A \to A
$$

$$
h(X) : X \mapsto \begin{cases}
\varnothing & [X=\varnothing] \\
X\setminus\{(c_X(\varnothing),\varnothing)\} & [\varnothing\in a(X)] \\
X & [\varnothing\notin a(X)]
\end{cases}
$$

$$
\varphi : A \to \mathrm{Ord}
$$

$$
\varphi X : (X,\alpha) \mapsto \begin{cases}
1 & [X=\varnothing] \\
\omega^{c_X(\varnothing)} & [a(X)=\{\varnothing\}] \\
\mathrm{enum}(\{\alpha \mid \forall\beta < f(h(X))[\alpha = \varphi d(g(h(X),\beta)\cup\{(\alpha,\varnothing)\})]\})(c_X(\varnothing)) & [f(h(X))>2] \\
\mathrm{enum}(\{\alpha \mid \alpha = \varphi g(h(X),\alpha)\})(c_X(\varnothing)) & [f(h(X))=2]
\end{cases}
$$

$$
\begin{array}{rcl}
\underline{\alpha} & = & \{(\alpha,0)\} \\
\{(\alpha_i,\underline{i})\}_{i\in n,n\in\omega} & = & (\alpha_i)_{i\in n,n\in\omega} \\
\{(\alpha_i,X_i)\}_{i\in n,n\in\omega} & = & \begin{pmatrix}\alpha_i \\ X_i\end{pmatrix}_{i\in n,n\in\omega}
\end{array}
$$